Int. Journal of Management and Development Studies 6(3): 5-16 (2017)
ISSN (Online): 2320-0685. ISSN (Print): 2321-1423
Impact Factor: 0.715

A Multi-Vendor Single-Buyer Integrated Inventory Model with Shortages

Debashish Kumar[1], D. A. Khan[2] and T. Mandal[3]

Abstract: *This paper develops a more general production-inventory model for a one-buyer-multi-vendor integrated system. We consider the problems where a manufacturer faces problem from the supply side when sourcing a product from heterogeneous suppliers. The model is extended to the situation with shortage permitted, based on shortage being allowed to occur only for the buyer. A numerical example and sensitivity analysis is presented to illustrate the procedure.*

Keywords: Inventory, Inventory system, integrated inventory, shortages.

Introduction

In today's competitive environment, organizations are forced to optimize their business operations to meet the increasing customer's demand within due time keeping desired quality level in minimum cost. A closer look at the literature reveals that previous research focused on single-vendor-single-buyer and single-vendor-multiple-buyer models, which predominantly focused on the sales side of the supply chain. The problem that arise on the supply side have often been considered in multi echelon integrated inventory model like multi-vendor-single-buyer or multi-vendor-multi-buyer taking into consideration of either single-product or multi-product. There are only few researches focused on multi-vendor-single-buyer models like automobile industries.

A closer look at the literature reveals that the relationships of the manufacturer to its suppliers also need to be studied, and that the problem of finding delivery times and lot sizes (or quantities) in a n:1-scenarion is not necessarily the same companies face when coordinating deliveries from a manufacturer to multiple customers i.e. 1:n-scenarion. The supplier inventory carrying cost reduce when frequently deliver to the buyer, but that result in high transportation cost especially when both type of parties not located to each other. On the other hand if supplier ships large quantities to reduce the overall number of shipments in a cycle, which leads to high inventory carrying cost for both parties [Glock and Kim (2014)].

In a multi-vendor-single-buyer scenario, the organization has to select the supplier from a set of pre-selected heterogeneous supplier and when the supplier capacity, quality level, lead time and various cost parameters like unit holding cost, transportation cost etc. also vary. None of the suppliers is able to satisfy the organization's total demand due to various limitations at the supplier's end such as its capacity, quality level, delivery time, price etc. Supplier capacity differ from time to time due to their own internal or external issues. There are rare chances of shortage

[1] Information Technology, XLRI Jamshedpur, Jharkhand, India.
[2] Department of Computer Applications, NIT Jamshedpur, Jharkhand, India.
[3] Department of Mathematics, NIT Jamshedpur, Jharkhand, India
Corresponding author: Debashish Kumar can be contacted at: dkumar@xlri.ac.in
Any remaining errors or omissions rest solely with the author(s) of this paper

in case of multiple suppliers, but may occurs due to uncertainties in the form of demand from market, capacity of supplier, uncertainty in supplier lead time, quality uncertainty and production failure in supplier end [Ware et al. (2014)].

Inventory systems with shortage of raw materials have received considerable attention in recent years. These are systems in which item(s) short in stock due to either bad coordination among suppliers or increase in demand compare to production rate of supplier(s), experience continuous shortage over time. In this paper, author shall be concerned with the problem of finding the optimal replenishment policy for an inventory system with shortages, in which products are continuously short over time at a constant rate and demand rates are constant over a known and finite planning horizon.

Glock [2011] model which can be seen as a special case of the modal treated in this paper. He consider a situation where the buyer faces a pool of heterogeneous suppliers, and tackles the supplier selection and lot size decision with the objective to minimize the system costs. In this paper, the procedure introduced by Glock [2011] is extended to the situation with shortages permitted to occur. Under the assumptions that shortages are allowed to occur only for buyer and that the allowable shortage amount between any two successive shipments are the same, the paper present a deterministic model with shortages for the vendor-buyer integrated system to find the optimal production and shipment policy. A related procedure was suggested by Zhou and Wang [2007] for the case of shortages for a single-vendor-single-buyer integrated system.

The remainder of this paper is organized as follows: The next two sections give an overview of related works and outline the assumptions and definitions that will be used in the remaining parts of this paper. Section 4 presents the model formulation for a one-buyer multi-vendor integrated inventory problem with shortage permitted. Section 5 presents a numerical example and sensitivity analysis and section 6 states conclusion and future scope.

Literature Review

The one-supplier one-buyer integrated inventory model is generally considered as the basic building block of every supply chain. One of the first lot-size model with single actor on each echelon dealing with buyer-vendor coordination was proposed by Goyal [1976], who analyzed a system wherein a single vendor delivers a product in equal-size shipments to a buyer. He assumed that the vendor acts as a reseller of the product and provide an infinite replenishment rate. Goyal [1977] suggested a joint economic lot size model where a collaborative arrangement between the buyer and the vendor is enforced by some contractual agreement and the inventory holding costs are independent of the price of the item. Goyal's model is generalized by Banerjee [1986], who considers a finite production rate at the vendor and thus analyzed where the vendor acts as a manufacturer who produces to order for the buyer. Goyal [1988] further generalized Banerjee's model by relaxing the assumption of the lot-for-lot policy of the vendor and showed that the vendor's lot size is an integer multiple of the buyer's order size provides a lower or equal joint total relevant cost. Goyal [1988] and Lu [1995] assumed that the items are sent to the buyer in equal sized shipments, whereas Goyal [1995] suggested that the *i*th shipment size to the buyer should be determined by evaluating

(First shipment size) * (Production rate/Demand rate)$^{(i\text{-}1)}$

Hill [1997] studied the impact of alternative shipment policies and showed that cost can be reduced in many cases if production quantities are delivered in multiple deliveries to the

buyer. Hill [1999] showed that the optimal policy in a single-vendor-single-buyer model is to ship batches that increase in size by a fixed factor, followed by equal-sized shipments.
Lu [1995] and Hill [1997, 1999] have extended the basic integrated inventory models and assume that the vendor is able to deliver an integrate consumption of a shipments per production cycle to the buyer. Thus, the buyer may initiate consumption of a lot earlier, which reduces minimum inventory in the system and consequently leads to lower inventory carrying costs. Hoque [2008] extended a close relationship between manufacturer and buyers for a costless way of benefit sharing. He introduced three models, two of which transfer with equal batches and the third with unequal batches of the product. Leopoldo [2001] introduced an algebraic method to solve the classic economic order quantity (EOQ) and economic production quantity (EPQ) model without the use of derivatives.

Suppliers are the vendors who provide raw materials, components or services that an organization itself cannot offer. Selection of appropriate suppliers is one of the fundamental strategies for enhancing the product quality of any organization. In the current manufacturing environment for supply chains, the right supplier can furnish a company with quality products of required quantity at reasonable prices before the predetermined delivery schedule. There have been number of methods proposed in supplier selection based on the different evaluation indicators like quality, delivery schedule and past performance (Lehmann and O'Shaughnessy [1982]), supplier technological capacity, financing capability, after-sale service (Goffin et al. [1997]), supplier capability and performance (Narasimhan et al. [2001]), green competencies, the state of natural resources, supplier's green image and net life cycle cost (Noci [1997]) and many more.

In supplier selection problem, quantitative models mainly focus on the question of which vendor to select and how to allocate the order quantity to the suppliers. Kim and Goyal [2009] who study an integrated model with a single buyer and multiple suppliers. The author compare two different delivery structure – where all suppliers deliver their respective production lots simultaneously or the suppliers deliver successively. Another model is proposed by Hong and Hayya [1992], who consider a just-in-time scenario wherein the buyer intends to reduce his lot size, either by spitting a larger order into multiple deliveries or by allocating the order quantity to multiple suppliers. Glock [2011] tackled both the supplier selection and the lot-size decision problem in a multi-vendors single-buyer environment, where the buyer sources a product from heterogeneous suppliers and the objective is to minimize the total system costs. Gock [2012] presented a comprehensive review of the literature published in joint economic lot size (JLES) models and precisely categorize and synthesize existing works in this area.

The intension of this paper is to study shipment consolidation in an integrated inventory model, which assumes that an individual supplier is not able to satisfy the entire demand at the buyer causes shortages. It assumes a buyer who replenishes into inventory from multiple vendors, where vendors deliver their respective production which may occur by avoiding the delivery cycle overlap.

Model assumptions and notations

The following assumptions and notations are used to develop the model.

Assumptions

This article considers a single buyer who orders a single product at multiple vendors.
The main objective of this article is to minimize the total system costs under study.

- All parameters are deterministic and constant over time
- The suppliers are heterogeneous, i.e. their cost parameter may be different.
- The production capacity of individual supplier among the preselected supplier can be smaller than the demand rate of the buyer. However, the sum of the production capacities of all pre-selected suppliers may or may not be always larger than demand rate of the buyer.
- Shortage are allowed to occur and to backlog completely only for the buyer.
- A lot produced at one of the vendors is sent in equal-sized batch shipments to the buyer. However, the no. of batch shipments may be vary for each vendor.
- Due to the value added concept, we assume that the unit inventory carrying charges per unit of time at the buyer h^b are higher than the unit inventory carrying charges per unit of time at each vendor, h^v.
- The buyer holding cost lower than the vendor's is more beneficial to the system if shortages are not permitted to occur, otherwise it just reverses.
- Co-ordination between the vendor and buyer
- The allowable shortage amounts for buyer between any two successive shipments are the same.

Notations

A Ordering cost per order
S_i Setup costs per setup at vendor i
C_i Cost per unit ordered at vendor i
R Relationship management costs per supplier
F_i Transportation costs per delivery at vendor i
D Demand rate at the buyer
h^b Unit inventory carrying charges per unit of time at the buyer
h^v Unit inventory carrying charges per unit of time at the vendor i
P_i Production rate at vendor i
N Number of vendors in the supplier pool, with n no. of shipment all together
β_i Proportion of the order lot size that is produced by vendor i with $\sum_{i=1}^{N} \beta i = 1$, where N is the number of vendor in supplier pool [decision variable]
Q Order lot size with $Q = \sum_{i=1}^{N} q_i$, where q_i is the production lot size of the vendor i [decision variable]
m_i Number of equal-sized batches per lot of vendor i [decision variable]
q_i Production lot size of vendor i with $q_i = \beta Q$
S_b Shortage cost per unit per unit time for the buyer
S allowable shortage amount for the buyer between two successive shipments
t_s time that builds a backorder level of S units
T_i time gap from i^{th} to $i+1^{th}$ shipment

The model and its solution

This paper consist of an inventory problem, where the lot size Q, the individual production quantities q_i and shipment frequency m_i have to be calculate with the objective to minimize total system costs when shortage occur and skip the selection procedure of supplier from the set of pre-selected supplier.

Vendor's Average Cost

The vendor's average stock under shortage case is also same as without shortage (Zhou and Wang [2007]). The time weighted inventory [$TWI_i^{(v)}$] can be calculated by computing the cumulative production quantity of the vendor in a production cycle and subtracting the cumulative quantity shipped to the buyer (Appendix A of Glock [2011]).

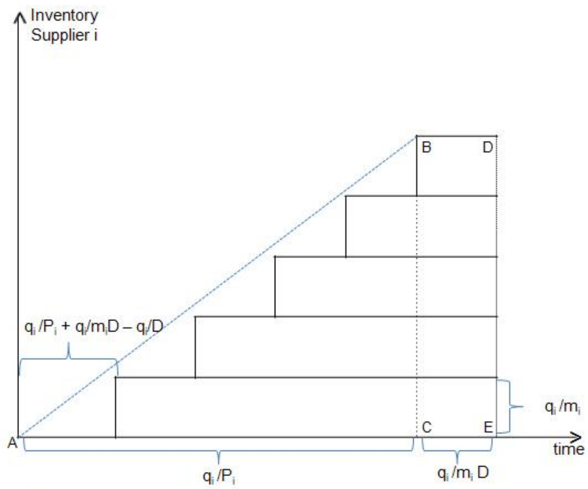

Figure 1. Cumulative production & shipments of a vendor in a production cycle when D>P_i

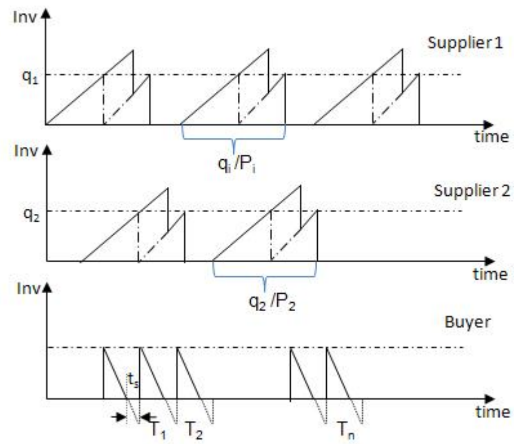

Figure 2. Stock against time for a two suppliers and a single buyer

Fig. 1 illustrates the cumulative quantity produced and shipped for a vendor for the case $D > P_i$. To avoid shortage in case of $D > P_i$, the vendor has to keep the first batches in stock for some additional time after they have been completed to assure that the demand at the buyer may be satisfied without interruption. As a consequence, the first batch is shipped after $\frac{q_i}{P_i} + \frac{q_i}{(m_i D)} - \frac{q_i}{D}$ time unit and the remaining batches every $\frac{q_i}{(m_i D)}$ time afterwards.

The cumulative production quantity represents the triangle ABC and is thus given as: $\frac{q_i^2}{2P_i}$

The cumulative quantity shipped is equivalent to the step-ladder, reduced by the rectangle BCED. The time-weighted inventory thus equals:

$$TWI_i^{(v)} = q_i^2 \left(\frac{1}{2P_i} - \frac{m_i - 1}{2m_i D}\right) \quad (1)$$

The inventory carrying cost [$IC^{(v)}$] per unit of time can be calculated if (1) is divided by cycle time and multiplied with the inventory carrying charges per unit of time i.e. $h_i^{(v)}$.

$$IC^{(v)} = q_i^2 \left(\frac{1}{2P_i} - \frac{m_i - 1}{2m_i D}\right) \frac{Dh_i^{(v)}}{Q} \quad (2)$$

Apart from inventory carrying cost, vendor i encounters setup costs per setup S_i and transportation cost per batch shipped to the buyer. Further, the vendor has to bear production costs for each unit produced, which amount to $C_i q_i$ for a production lot. The total cost of vendor i are thus given as:

$$TC_i^{(v)} = q_i^2\left(\frac{1}{2P_i} - \frac{m_i - 1}{2m_i D}\right)\frac{D h_i^v}{Q} + (S_i + F_i m_i + C_i q_i)\frac{D}{Q} \quad (3)$$

Buyer's average cost

Consider an integrated production-inventory system in which N numbers of pre-selected vendors produces a single item and supplies it to a buyer. If q_i production lot size of the vendor i is transported to the buyer in m_i shipments (equal-sized batches), the problem is how to determine an order lot size,Q $(=\sum_{i=1}^{N} q_i)$, and a shipment policy that minimize the average total cost of the integrated system.

Since shortages are allowed to occur, the stock positions associated with this integrated system can be illustrated in figure 2. Assume that the buyer orders a lot size Q, which is allocated to the vendors in such a way that q_i equals the production lot size of supplier i with q_i=$\beta_i Q$

It is necessary to restrict the lot q_i in size is order to assure that the time which is necessary to produce a lot of size q_i is larger than the time needed to consume one lot of each vendor at the buyer. Thus it follows that

$$\sum \frac{q_i}{D} \geq \frac{q_i}{P_i}$$

The time weighted inventory for the buyer is given by

$$TWI^{(b)} = D[(T_1 - t_s)^2 + (T_2 - t_s)^2 + (T_3 - t_s)^2 + \cdots + (T_k - t_s)^2 + \cdots + (T_n - t_s)^2]/2$$
$$= D\sum(T_i - t_s)^2/2 \quad (4)$$

Therefore, the average buyer inventory cost is

$$IC^{(b)} = \left[TWI^{(b)}/Cycle\ Time\right] \times h^b$$
$$= [D^2\sum(T_i - t_s)^2/2Q] \times h^b$$
$$= D^2 h^b \sum(T_i - t_s)^2/2Q \quad (5)$$

Where, $T_i = q_i/Dm_i$

Similarly, the average shortage level for the buyer is $nDt_s^2/2$

And thus the average shortage cost for the buyer is

$$SC^{(b)} = [nD^2 t_s^2/2Q] \times S_b \quad (6)$$

Thus the total cost of the buyer is

$TC^{(b)}$ = Average buyer inventory cost ($IC^{(b)}$) + Fixed ordering cost + Cost of Relationship Management + Shortage Cost

$$TC^{(b)} = \frac{D^2 h^b \sum(T_i - t_s)^2}{2Q} + \frac{AD}{Q} + R\sum_{i=1}^{N}\delta_i + \frac{nD^2 t_s^2 S_b}{2Q} \quad (7)$$

System cost

The total cost of the system may now be calculated as the sum of $TC_i^{(v)}$ and $TC^{(b)}$, is given by

$$TC^{(s)} = q_i^2\left(\frac{1}{2P_i} - \frac{m_i - 1}{2m_i D}\right)\frac{D h_i^{(v)}}{Q} + \sum(S_i + F_i m_i + C_i q_i)\frac{D}{Q} + \frac{D^2 h^b \sum(T_i - t_s)^2}{2Q} + \frac{AD}{Q} + R\sum_{i=1}^{N}\delta_i + \frac{nD^2 t_s^2 S_b}{2Q}$$

Assume $\lambda_i = \frac{1}{2P_i} - \frac{m_i - 1}{2m_i D}$ and putting $q_i = \beta_i Q$ in above equation, we get

$$TC^{(s)} = Dh_i^{(v)} Q \sum \beta_i^2 \lambda_i + \frac{D}{Q}\left(\sum(S_i + F_i m_i) + A\right) + \sum C_i \beta_i D + \frac{D^2 h^b \sum(T_i - t_s)^2}{2Q} + R\sum_{i=1}^{N} \delta_i + \frac{nD^2 t_s^2 S_b}{2Q} \quad (8)$$

From (8) one can easily know that $\partial^2 TC^{(s)}/\partial {t_s}^2 > 0$ [see Appendix A]. Hence, solving $\partial TC^{(s)}/\partial t_s = 0$ will yield the optimal value of t_s as

$$t_s^* = \frac{h^b \sum T_i}{n(h^b + S_b)} \quad (9)$$

Substituting (9) into (8) will give the total cost as

$$TC^{(s)} = DQ\sum \beta_i^2 \lambda_i h_i^{(v)} + \frac{D}{Q}\left(\sum(S_i + F_i m_i) + A\right) + D\sum C_i \beta_i + \frac{D^2 h^b}{2Q}\left(\sum T_i - \frac{h^b \sum T_i}{n(h^b + S_b)}\right)^2 + R\sum_{i=1}^{N} \delta_i + \frac{nD^2 S_b}{2Q}\left(\frac{h^b \sum T_i}{n(h^b + S_b)}\right)^2 \quad (10)$$

From (10) one can easily know that $\partial^2 TC^{(s)}/\partial Q^2 > 0$ [see Appendix B]. Hence, solving $\partial TC^{(s)}/\partial Q = 0$ will yield the optimal value of Q as

$$Q = \sqrt{\frac{D\sum(S_i + F_i m_i) + DA + \frac{D^2 h^b}{2}\left(\sum T_i - \frac{h^b \sum T_i}{n(h^b + S_b)}\right)^2 + \frac{nD^2 S_b}{2}\left(\frac{h^b \sum T_i}{n(h^b + S_b)}\right)^2}{D\sum \beta_i^2 \lambda_i h_i^v}}$$

$$= \sqrt{\frac{\chi}{D\sum \beta_i^2 \lambda_i h_i^v}} \quad (11)$$

Where $\chi = D\sum(S_i + F_i m_i) + DA + \frac{D^2 h^{(b)}}{2}\left(\sum T_i - \frac{h^b \sum T_i}{n(h^b + S_b)}\right)^2 + \frac{nD^2 S_b}{2}\left(\frac{h^b \sum T_i}{n(h^b + S_b)}\right)^2$

Substituting (11) into (10) will give the total cost as

$$TC^{(s)} = DQ\sum \beta_i^2 \lambda_i h_i^{(v)} + \frac{\chi}{Q} + D\sum C_i \beta_i + R\sum_{i=1}^{N} \delta_i$$

$$= D\sum \beta_i^2 \lambda_i h_i^v \sqrt{\frac{\chi}{D\sum \beta_i^2 \lambda_i h_i^v}} + \chi\sqrt{\frac{D\sum \beta_i^2 \lambda_i h_i^v}{\chi}} + D\sum C_i \beta_i + R\sum_{i=1}^{N} \delta_i$$

$$= \sqrt{\chi D\sum \beta_i^2 \lambda_i h_i^v} + \sqrt{\chi D\sum \beta_i^2 \lambda_i h_i^v} + D\sum C_i \beta_i + R\sum_{i=1}^{N} \delta_i$$

$$= 2\sqrt{\chi D\sum \beta_i^2 \lambda_i h_i^v} + D\sum C_i \beta_i + R\sum_{i=1}^{N} \delta_i \quad (12)$$

Numerical example

To illustrate the model developed in this article, we considered the three and four different pre-selected vendors data as given in Table 1 and vendor selection criteria is not consider in this article and random selection is taking care based on vendor's production rate, setup costs and transportation costs. We consider the parameter values D=150, A=100 and S_b=1.5 in the appropriate units and solve all the test problem for $h^{(b)}$=5 and R=60.

The results of this are presented in Table 2. The second column of Table 2 illustrates how many vendors were selected in the final solution.

Table 1: Vendor Data Solved In the Numerical Study

S. No	N	$\{P_1...P_n\}$	$\{h^v_1...h^v_n\}$	$\{C_1...C_n\}$	$\{S_1...S_n\}$	$\{F_1...F_n\}$
1	3	(60.155,90)	(1,2,2)	(1,2,3)	(150,250,230)	(55,60,105)
2	4	(75,100,155,80)	(2,1,4,3)	(1,2,3,2)	(40,40,75,50)	(70,40,55,35)

Table 2: Optimal solution for $h^{(b)=}$5 and R=60

S. No	Sel	Q	$\{\beta_1...\beta_n\}$	$\{m_1...m_n\}$	t_s	S	TC(s)
1	2	1024.88	(0.4,0.6,0)	(4,6,0)	0.05	7.69	746.15
2	1	1717.58	(0,1.0,0)	(0,10,0)	0.03	3.85	652.70
3	2	736.05	(0.4,0,0.6)	(4,0,6)	0.05	7.69	1037.72
4	2	661.99	(0.33,0.67,0,0)	(4,6,0,0)	0.04	5.60	748.84
5	2	735.00	(0.33,0,0.67,0)	(4,0,6,0)	0.04	5.60	848.49
6	3	446.08	(0.33,0.37,0,0.3)	(4,2.2,0,1.8)	0.05	7.23	956.08

Sensitivity Analysis

We now study sensitivity of the optimal solution for the case $h^{(b)}$=5 and R=60 to changes in the values of only one of the parameters (C_i, S_i, F_i, $h^{(b)}$, R and S_b) by (+50%, +25%, -25% and -50%). The sensitivity analysis is shown in Table 3 (for #1 solution in Table 2) and Table 4 (for #4 solution of Table 2). The following points are observed:

1. Total cost (TC) is slightly sensitive to change in C_i, F_i, $h^{(b)}$, R, S_b and $h^{(v)}$ while moderately sensitive to change in S_i, D and P. This means that the setup cost (S_i), demand rate (D) and the production rate (P) should be estimated carefully to determine the optimal system cost correctly.
2. Allowable shortage amount (S) has no sensitivity to change in the parameters C_i, S_i, F_i,, R, $h^{(v)}$ and P while little sensitive to changes in $h^{(b)}$ and D and its sensitivity to negative errors in shortage cost (S_b) gradually becomes higher. Larger negative errors in S_b imply that penalties due to shortage in inventory decrease considerably.
3.

Table 3: Sensitivity Analysis for the Optimal Solution in the Case of Three Vendors

Parameter	*% change*	*Quantity order (% change)*	*Total Cost (TC) (% change)*	*Shortage Time (t_s) (% change)*	*Shortage (S) (% change)*
C_i	+50	-	+16.08	-	-
	+25	-	+8.04	-	-
	-25	-	-8.04	-	-
	-50	-	-16.08	-	-
S_i	+50	+11.3	+5.85	-	-
	+25	+5.8	+3	-	-
	-25	-6.16	-3.19	-	-
	-50	-12.75	-6.6	-	-
F_i	+50	+10.45	+5.41	-	-
	+25	+5.35	+2.77	-	-
	-25	-5.66	-2.93	-	-
	-50	-11.67	-6.04	-	-

h(b)	+50	-0.01	-0.01	+8.33	+8.33
	+25	-0.01	0	+4.84	+4.84
	-25	+0.01	+0.01	-7.14	-7.14
	-50	+0.03	+0.03	-18.75	-18.75
R	+50	-	+8.04	-	-
	+25	-	+4.02	-	-
	-25	-	-4.02	-	-
	-50	-	-8.04	-	-
S_b	+50	+0.03	+0.02	-10.34	-10.34
	+25	+0.02	+0.01	-5.45	-5.45
	-25	-0.02	-0.01	+6.12	+6.12
	-50	-0.04	-0.02	+13.04	+13.04
h(v)	+50	-18.35	+11.63	-	-
	+25	-10.56	+6.11	-	-
	-25	+15.47	-6.93	-	-
	-50	+41.42	-15.16	-	-
D	+50	-21.71	+63.82	-	+50
	+25	-14.87	+32.41	-	+25
	-25	+65.83	-36.43	-	-25
	-50			-	-50
P	+50	+480.45	-42.84	-	-
	+25	+54.71	-18.3	-	-
	-25	-28.76	+20.89	-	-
	-50	-49.43	+50.59	-	-

Table 4: Sensitivity Analysis for the Optimal Solution in the Case of Four Vendors

Parameter	*% change*	*Quantity order (% change)*	*Total Cost (TC) (% change)*	*Shortage Time (t_s) (% change)*	*Shortage (S) (% change)*
C_i	+50	-	+16.73	-	-
	+25	-	+8.36	-	-
	-25	-	-8.36	-	-
	-50	-	-16.73	-	-
S_i	+50	+5.96	+3.01	-	-
	+25	+3.02	+1.53	-	-
	-25	-3.12	-1.58	-	-
	-50	-6.34	-3.2	-	-
F_i	+50	+14.52	+7.33	-	-
	+25	+7.5	+3.79	-	-
	-25	-8.12	-4.1	-	-
	-50	-17.02	-8.6	-	-
h(b)	+50	+0.25	+0.13	+8.33	+8.33
	+25	+0.13	+0.06	+4.84	+4.84
	-25	-0.13	-0.07	-7.14	-7.14
	-50	-0.26	-0.13	-18.75	-18.75
R	+50	-	+8.01	-	-

	+25	-	+4.01	-	-
	-25	-	-4.01	-	-
	-50	-	-8.01	-	-
S_b	+50	+0.04	+0.02	-10.34	-10.34
	+25	+0.02	+0.01	-5.45	-5.45
	-25	-0.02	-0.01	+6.12	+6.12
	-50	-0.05	-0.02	+13.04	+13.04
h(v)	+50	-18.35	+11.35	-	-
	+25	-10.56	+5.96	-	-
	-25	+15.47	-6.77	-	-
	-50	+41.42	-14.8	-	-
D	+50	-12.49	+53.31	-	+50
	+25	-8.12	+26.78	-	+25
	-25	+20.51	-27.54	-	-25
	-50	+308.28	-61.1	-	-50
P	+50	+68.26	-20.5	-	-
	+25	+27.84	-11	-	-
	-25	-22.07	+14.31	-	-
	-50	-41.68	+36.11	-	-

Conclusion

In this paper, we reconsider the shipment problem when a single buyer sourcing a single product from multiple heterogeneous suppliers when shortages occur and tackled the lot size decision with the objective to minimize total system costs. The supplier selection decision is precisely discussed by Glock [2011] and skips in this article.

The model can further extended to some more practical situations, such as considering supplier selection decision/algorithm, permitting a variable production rate at the vendors, multiple items, multiple-vendor multiple-buyer planning situation, introduce bargaining power into the model etc. The allowable shortage amounts for buyer between any two successive shipments vary i.e. multi period case. We will consider these problems in the near future.

Appendix A

Differentiating equation (8) w.r.t. ∂t_s

$$\partial TC^{(s)}/\partial t_s = 0 + 0 + 0 + \frac{D^2 h^b}{2Q}\sum(0 - 2T_i + 2nt_s) + \frac{nD^2 t_s S_b}{Q}$$

$$= \frac{-D^2 h^b}{Q}\sum T_i + \frac{D^2 h^b n t_s}{Q} + \frac{nD^2 t_s S_b}{Q}$$

$$= \frac{-D^2 h^b}{Q}\sum T_i + \frac{D^2 n t_s}{Q}(h^b + S_b)$$

Since $\partial^2 TC^{(s)}/\partial {t_s}^2 = \frac{D^2 n}{Q}(h^b + S_b) > 0$ always

$\therefore\ \partial TC^{(s)}/\partial t_s = 0$

$$t_s = \frac{h^b \sum T_i}{n(h^b + S_b)}$$

Appendix B

Assume $\chi = D\sum(S_i + F_i m_i) + DA + \frac{D^2 h^b}{2}\left(\sum T_i - \frac{h^b \sum T_i}{n(h^b + S_b)}\right)^2 + \frac{nD^2 S_b}{2}\left(\frac{h^b \sum T_i}{n(h^b + S_b)}\right)^2$

Equation (10) can be rewrite as

$$TC^{(s)} = DQ\beta_i^2 \lambda_i h_i^v + \frac{\chi}{Q} + D\sum C_i \beta_i + R\sum_{i=1}^{N} \delta_i \quad \text{B.1}$$

Differentiating equation (B.1) w.r.t. ∂Q, we get

$$\partial TC^{(s)}/\partial Q = D\beta_i^2 \lambda_i h_i^{(v)} - \frac{\chi}{Q^2} + 0 + 0 \quad \text{B.2}$$

Since $\partial^2 TC^{(s)}/\partial Q^2 = \frac{2\chi}{Q^3} > 0$ always

$\therefore \partial TC^{(s)}/\partial Q = 0$

$$Q = \sqrt{\frac{\chi}{D\sum \beta_i^2 \lambda_i h_i^{(v)}}} \quad \text{B.3}$$

References

Goyal S.K.: An integrated inventory model for a single supplier single customer problem. International Journal of Production Research, 14, 107-111 (1976)

Goyal S.K.: Determination of optimum production quality for a two-stage production system. Operation Research Quarterly, 28, 865-870 (1977)

Lehmann D.R. and O'Shaughnessy J.: Decision criteria used in buying different categories of products. Journal of Purchasing and Materials Management, Spring, 9-14 (1982)

Banerjee A.: A joint economic lot size model for purchaser and vendor. Decision Sciences, 17, 292-311 (1986)

Goyal S.K.: A joint economic lot size model for purchaser and vendor: A comment. Decision Sciences, 19, 236-241 (1988)

Hong J.D. & Hayya J.C.: Just-in-time purchasing: Single or multiple sourcing. International Journal of Production Economics, 27, 175-181 (1992)

Goyal S.K.: A one-vendor multi-buyer integrated inventory model: A comment. European Journal of Operational Research, 82, 209-210 (1995)

Lu L.: A one-vendor multi-buyer integrated inventory model. European Journal of Operational Research, 81, 312-323 (1995)

Hill Roger M.: The single-vendor single-buyer integrated production inventory model with a generalized policy. Europen Journal of Operational Research, 97, 493-499 (1997)

Goffin K., Szwejczewski M. and New C.: Managing suppliers: when fewer can mean more. International Journal of Physical Distribution and Logistic Management, 27 (7), 422-436 (1997)

Noci G.: Designing 'green' vendor rating systems for the assessment of supplier's environmental performance. European Journal of Purchasing and Supply Management, 3 (2), 103-114 (1997)

Hill Roger M.: The optimal production and shipment policy for the single-vendor single-buyer integrated inventory problem. International Journal of Production Research, 37, 2463-2475 (1999)

Cardenas-Barron Leopoldo Eduardo: The economic production quantity (EOQ) with shortage derived algebraically. International Journal of Production Research, 50, 2910-2924 (2001)

Narasimhan R., Talluri S. and Mendez D.: Supplier evaluation and rationalization via data envelopment analysis an empirical examination. Supply Chain Management: An International Journal, 37 (3), 28-37 (2001)

Zhou Yong-Wu & Wang Sheng-Dong: Optimal production and shipment models for a single-vendor-single-buyer integrated system. European Journal of Operational Research, 180, 309-328 (2007)

Hoque M.A.: Synchronization in the single-manufacturer multi-buyer integrated inventory supply chain. European Journal of Operational Research, 188, 811-825 (2008)

Kim T. & Goyal S.K.: A consolidated delivery policy of multiple suppliers for a single buyer. International Journal of Procurement Management, 2, 267-287 (2009)

Glock Christoph H.: A multi-vendor single-buyer integrated inventory model with a variable number of vendors. Computer & Industrial Engineering, 60, 173-182 (2011)

Glock Christoph H.: The joint economic lot size problem: A review. International Journal of Production Economics, 135, 671-686 (2012)

Glock Christoph H. & Kim Taebok: Shipment consolidation in a multiple-vendor-single-buyer integrated inventory model. Computer & Industrial Engineering, 70, 31-42 (2014)

Ware Nilesh R., Singh S.P. & Banwet D.K.: A mixed-integer non-linear program to model dynamic supplier selection problem. Expert Systems with Applications, 41, 671-675 (2014)